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LETTER TO THE EDITOR

Strain-induced dynamics of flowing foam: an experimental study

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Abstract. Diffusing-wave spectroscopy has been used to study the effects of strain upon the dynamics of a model foam. Above a threshold strain, the rate at which elementary strain-induced rearrangements occur in the foam is proportional to strain rate. These results are in accord with the findings of recent computer simulations.

Foams are beautiful and mysterious [1, 2]: the beauty is apparent to every child; the mystery largely derives from the difficulty of studying such complex and disordered structures. Amongst their more important properties, from a practical view-point, are their mechanical response [3]. At low stress, foam behaves as an elastic solid, while above a reasonably well-defined yield stress it deforms plastically. This microscopic behaviour must involve local rearrangements of the bubbles comprising the foam. Until very recently such elementary rearrangements have been experimentally unobservable, except for the two-dimensional case. This letter presents the first experimental study, using the recently developed technique of diffusing wave spectroscopy (DWS) [4, 5], of the dependence of the rate of such rearrangements upon strain and strain rate in a flowing foam.

Much of our understanding of foam rheology rests upon computer simulations [6]. In particular the realistic situation of a disordered froth cannot, at least to date, be addressed analytically. In recent years simulations have become more realistic, extending to encompass not only disorder [7, 8, 9, 10], but also the inclusion of a liquid fraction in the foam [11]. Plastic deformation is found to occur via a succession of localized elementary events in which a group of bubbles rearrange to relieve local stress. In simulations of ideal dry froth (volume fraction of gas $\phi = 1.0$), elementary rearrangements occur when the imposed strain exceeds a value of about 0.5, corresponding to the yield stress [8]. Continued application of the same stress leads to indefinite increase of strain: the foam flows. Reduction of the gas fraction causes the critical value of strain to fall; it may reach zero at a critical ϕ ($= 0.84$) [11]. At a given strain above the critical value, a rearrangement event will occur at a particular point in the foam whenever the accumulated stress there exceeds some critical level. The rate of rearrangements should thus be proportional to the strain rate [10].

Foam is a highly multiply scattering material in which light can be regarded as propagating by diffusion [4]. The usual interference between light following several paths through the material gives rise to speckle, fluctuations in the interference terms causing the speckle to flicker, leading to spectral broadening of the scattered light. The phase change of $\approx \pi$ necessary to cause a change from constructive to destructive interference is here accumulated in events distributed over the *entire* light path, rather than in a single event as in

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conventional dynamic light scattering [12]. DWS allows spatially or temporally rare events to be observed; it is this aspect which is exploited in studies of foam [13, 14, 15, 16]. For static foam, it was concluded that a fundamental process involved in the temporal evolution of the foam was an elementary rearrangement event: a local neighbourhood some ten bubbles across spontaneously reorders as coarsening causes a critical value of the local stress to be exceeded [13, 15].

The material used in our experiments was Gillette shaving foam [17], a complex mixture comprising an aqueous solution of triethanolamine stearate, with smaller quantities of sodium lauryl sulphate and polyethylene glycol-(23) lauryl ether, together with hydrocarbon gas as a propellant. This material produces stable and reproducible foam samples, with $\phi = 0.92 \pm 0.01$ [13].

Our foam flow cell has been described elsewhere [16]. It was 55 cm long, comprising two glass plates separated by 5 mm wide aluminium spacers shaped to form a parallel-sided, 5 cm high channel narrowing in the centre as sketched in figure 1. The cell was sealed by two PTFE barriers, mechanically connected so that they could be moved in parallel, pushing the foam along the cell. In use, one barrier was removed, the cell was filled with foam, care being taken to exclude air, and the barrier was replaced to seal the cell. Barrier motion was induced with the pusher mechanism of an infusion pump, various speeds < 0.1 cm/min being used (these speeds are referred to below as the flow rate V).

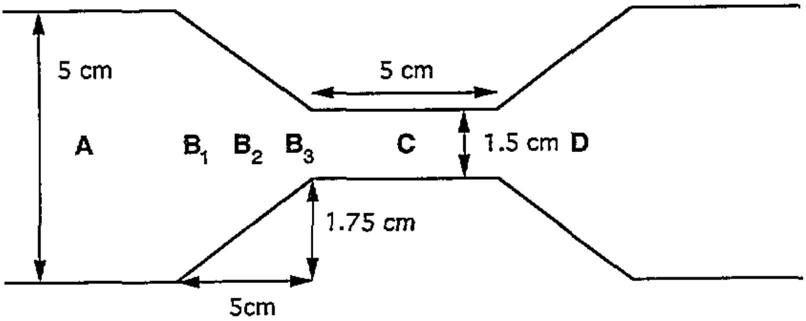

Figure 1. A sketch (not to scale) of the central portion of the flow cell used. DWS observations were made at various points in the regions A, B, C and D. See the text for further description.

A laser beam was normally incident on the cell and a few coherence areas of light forward scattered at 0° were selected for detection by a photomultiplier. From the measured intensity autocorrelation function of the scattered light the field correlation function, $g_T^{(1)}(\tau)$, was extracted. Observations were made at various points on the centre-line of the flow cell. Two points (A and C in figure 1) were in regions of unstrained foam, C being at the mid-point of the constriction. In region B of compressive strain, three points were studied: B_1 and B_3 some 3 mm from either end of the taper and the mid-point B_2. In region D of extensional strain, a single point half way along the taper was studied. These positions are inherently somewhat imprecise due to the spatial divergence of the light beam as it propagates through the optically dense medium.

Here we concentrate upon DWS observations in foam subject to strain, in the tapered sections of the cell. For a sample subject to a continuous deformation, as here, the linear strain $\mathrm{d}l/l$ is less appropriate than the Hencky strain [18]:

$$\epsilon = \int_{l_0}^{l} \frac{\mathrm{d}l}{l} = \ln(l/l_0) \tag{1}$$

where l_0 is the original dimension of the sample. Here we simply refer to the Hencky strain as 'the strain'. Similarly we use the rate of change of the Hencky strain

$$\dot{\epsilon} = \frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}t} \tag{2}$$

for the strain rate. Values of strain and strain rate appropriate to the various points in the cell where DWS observations were made are summarized in table 1. For point D, l_0 is taken as the narrowest width of the cell, as the foam observed by DWS started out in region C.

Table 1. Hencky strain and rate of change of Hencky strain (in terms of flow rate V) at various observation points.

Position	Strain	Strain Rate
B_1	−0.04	$0.15V$
B_2	−0.43	$0.33V$
B_3	−1.07	$1.20V$
D	0.77	$0.33V$

Due to temporal evolution [13, 14] the behaviour of the foam is non-stationary. We thus studied the behaviour of foam at a single age. At the highest flow rates used, it was not possible to flow the foam continuously through the cell at a constant rate, due to the finite cell size (barrier motion < 15 cm). For uniformity, then, the same protocol was used in all cases: after the cell was filled and sealed, the foam was allowed to age for some 150 min, when flow was initiated at the chosen rate. DWS observations were started at 165 min age, data being acquired over 15 min. Each observation at a given flow rate and position derives from a separate sample of foam observed at this age.

The correlation functions observed ($g_T^{(1)}(\tau)$) for flowing foams were closely exponential in form. We report here the decay constant Γ_1 characterizing the initial decay of $g_T^{(1)}(\tau)$. Slower components, such as were found in an earlier study of flowing foam [16], were observed when the correlator sample time was appropriately long, but will be neglected in what follows. DWS observations were made for flow rates from 0.0012 to 0.097 cm/min. In some cases observations were repeated, the scatter of Γ_1 being typically 10%. In these cases the uncertainties were taken as the standard error on the mean; otherwise the much smaller error on Γ_1 from the data analysis was used.

At points A and C, Γ_1 did not depend upon the flow rate. The average values for these two points (16.6 ± 2.3 s^{-1} at A and 13.1 ± 1.7 s^{-1} at C) were not statistically different, despite the non-zero strain at C. Both values are comparable to those found in static foam at 180 min age [14]. Fluctuations in such low values of Γ_1 probably account for the differences observed between points A and C in earlier, less comprehensive experiments [16].

The Γ_1 data for all points studied in the tapered regions are plotted versus strain rate in figure 2. At the lowest strain rate the data ($\langle\Gamma_1\rangle = 22 \pm 2$ s^{-1}) are in reasonable accord with those for points A and C. Considering the data for points B_2 and B_3 first, it seems that Γ_1 remains relatively low for $\dot{\epsilon}$ below some strain-dependent critical value, above which it increases sharply, subsequently being proportional to $\dot{\epsilon}$. Moreover, the separate linear dependences are essentially in perfect accord: the straight line plotted in the figure represents a fit to the data for the four highest flow rates at both these points. The data from point D, corresponding to extensional strain, again lie on this same linear variation. Any change in behaviour at point D as $\dot{\epsilon}$ rises above some threshold is less than that apparent at B_2 and B_3. The data for point B_1 differ: at very low strain rates Γ_1 seems reasonably close

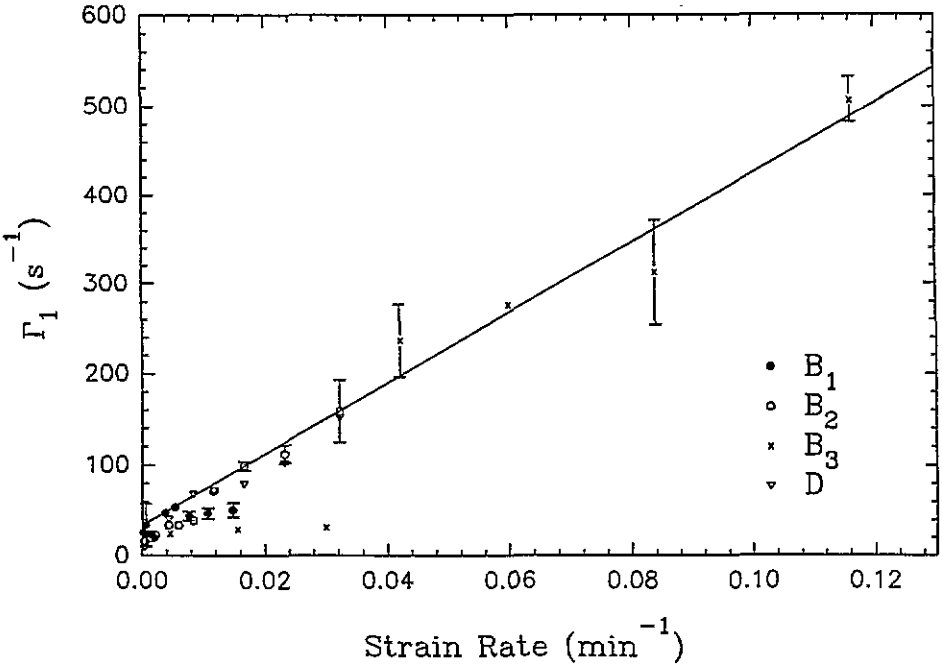

Figure 2. The dependence of the measured Γ_1 for points B_1 to B_3 and D upon strain rate. The line is a linear fit to certain of the data; see text for details.

to the extrapolated linear dependence just described, but thereafter it systematically falls away. This rather different behaviour at B_1 may indicate that ϵ at this point lies below the critical value at which strain-induced rearrangements occur in foam of gas volume fraction $\phi \approx 0.92$. While the data are suggestive, they do not provide a very stringent test, given the large increase in ϵ between B_1 and B_2 (table 1).

We do not here pursue the deviations from the linear dependence which seem to occur at low $\dot{\epsilon}$ for B_2 and B_3, but rather concentrate upon that linear dependence. It is apparent that for several different strains the initial decay of the DWS correlation functions simply depends linearly on strain rate over the whole range studied in the present experiments. The intercept of this linear fit, $33 \pm 17\ s^{-1}$, is consistent with the value determined for unstrained foam at A and C.

In the absence of a detailed theoretical understanding of the genesis of the DWS signal in foam, we turn to the arguments advanced by Durian *et al* [13], the generality of which suggests they should apply to the strained case equally as well as the static one. The approximately exponential $g_T^{(1)}(\tau)$ has initial decay constant

$$\Gamma_1 = \left(\frac{L}{l^*}\right)^2 \tau_0^{-1} \qquad (3)$$

which relates to the probability of a rearrangement event affecting light as it traverses the sample. L is the sample thickness and l^* the transport mean free path of the light, the average distance a photon travels before its propagation direction is randomized. The time τ_0 reflects the effect of the internal system dynamics upon the light scattering. For foam these dynamics take the form of spatially and temporally isolated rearrangement events in which the structure of a more or less extended group of cells changes when the local excess of stress exceeds some threshold value. It is suggested [13] that τ_0 reflects the average time interval between rearrangement events at any single point in the foam: $\tau_0 \approx Rr^3$ where R is the average rate of rearrangements per unit volume and r is the radius of the volume of foam involved directly in a typical event.

There is only one length scale in the system: the typical bubble size. This must set the scale of both l^* and r, although a single simple proportionality as assumed to date [13, 14] may not be exact under all circumstances. At all events, at a given foam age it seems reasonable to assume that the bubble size and hence l^* is constant: all samples were aged while static for ~ 150 min and so should have evolved similarly; any strain-dependent effects upon foam coarsening will only slightly affect d, being restricted to the final ~ 30 min of each experiment. Thus the strain-rate dependence of Γ_1 (figure 2) implies that τ_0^{-1}, the rate of occurrence of rearrangements at a given point in the foam, is simply linearly proportional to the strain rate.

This conclusion would be altered if l^* were affected by strain. For example, the relation between l^* and d might be changed by strain induced ordering [9] of an initially disordered foam. Such ordering presumably would only depend on ϵ, and not upon $\dot{\epsilon}$, and would thus have the same effect on all values of Γ_1 determined at a given point in the cell. The collapse onto a common variation of the linear dependences on strain rate for the different observation points (figure 2) and the accord of $\langle\Gamma_1\rangle$ for points A and C (where $\epsilon \neq 0$, $\dot{\epsilon} = 0$) appear to rule out such effects. While complex scenarios could be devised, the simplest conclusion is that figure 2 simply indicates that $\tau_0 \propto \dot{\epsilon}$.

In conclusion, while further work is needed to pursue some aspects of the present data, it is clear that the rate at which elementary bubble rearrangement events occur at any given point in foam subject to strain is just proportional to the strain rate. It appears that the yield strain for the present foam, having $\phi \approx 0.92$, exceeds 0.04, the strain at point B_1. These findings represent the first experimental confirmation of the results of recent computer simulations [8, 11, 10].

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